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Unified BRST Structure for Gravity and Supergravity

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Enlarging the gauge group with two extra fermionic coordinates, we provide a unified geometric formulation of the BRST and anti-BRST transformations for gravity in the vierbein formalism and simple supergravity.

1. INTRODUCTION

Since Faddeev–Popov ghosts are anticommuting fields and the generators of the BRST and anti-BRST transformations anticommute with each other (Becchi *et al.*, 1975; Tyutin, 1975; Curci and Ferrari, 1976; Ojima, 1980), it is clear that a suitable framework for analyzing the geometrical structure of a quantized gauge theory is differential supergeometry (Bonora and Tonin, 1981; Bonora *et al.*, 1982; Hoyos *et al.*, 1982).

In this geometrical setting for gauge theories with Faddeev–Popov fields the BRST and anti-BRST transformations are given by translations on the anticommuting coordinates of the (4, 2)-dimensional superspace M_s. On the other hand, it is interesting to observe that in this approach constraints should be imposed on the supercurvature corresponding to the superconnection in M_s, which contains more component fields than necessary to describe the physics. Appropriate constraints are given by imposing the vanishing of the supercurvature components along some anticommuting direction (soul-flatness condition). In the context of quantized Yang–Mills theories, these have been obtained as equations of motion by introducing a super-Lagrangian (Hoyos *et al.*, 1983).

In this superspace formulation of gauge theories, a prescription has been given in which the quantum Lagrangian for Yang–Mills theory, supersymmet-

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ric Yang–Mills theory, and gravity is derived (Hirschfeld and Leschke, 1981; Falck *et al.*, 1983; Falck and Hirschfeld, 1984).

General solutions to these supercurvature constraints in terms of unconstrained prepotential superfields for ordinary and supersymmetric gauge theories have been considered in Dur and Gates (1990) and Hull *et al.* (1991).

Recently, the BRST and anti-BRST transformations in Yang–Mills theory have been interpreted geometrically in terms of the superfiber bundle formalism (Loumi and Tahiri, 1993).

In the present paper, we propose to extend the geometrical formulation of Yang–Mills theory elaborated in Loumi and Tahiri (1993) to gravity in the vierbein formalism and to simple supergravity.

This paper is organized as follows: In Section 2 we provide a general geometric formulation of the BRST and anti-BRST symmetries. The basic idea is to enlarge with extra fermionic coordinates the gauge group of the original theory, be it a Yang–Mills, a gravity, or a supergravity theory. One of the results of this section will be to obtain the soul-flatness condition naturally from the geometrical structure without developing a super-Lagrangian theory as in the superspace formulation of BRST invariant Yang–Mills theories (Hoyos *et al.*, 1983). In Sections 3 and 4 we apply the general formalism of Section 2 to gravity in the vierbein formalism and to simple supergravity. Section 5 is devoted to discussion.

2. GENERAL FORMALISM

2.1. The Geometrical Scheme

Gauge theories are naturally described as geometrical theories over a principal (super) fiber bundle $P(M, G)$. The base space M and the structural group G are identified with space-time and gauge group, respectively. The gauge fields A_μ and the strength field tensor $F_{\mu\nu}$ are the coefficients of a connection ω and its curvature Ω, respectively.

In Yang–Mills theory, G is a d-dimensional internal compact Lie group. In gravity, G is the 10-dimensional Poincaré group P. In supergravity, G is the (10, 4)-dimensional super-Poincaré group SP (Yates, 1980).

When a gauge theory is quantized, one needs to introduce Faddeev–Popov fields so that the original gauge invariance is broken and new invariances arise, the BRST and anti-BRST invariances (Becchi *et al.*, 1975; Tyutin, 1975; Curci and Ferrari, 1976; Ojima, 1980).

The aim of this section is to find a geometrical formulation of BRST and anti-BRST transformations where the soul-flatness condition is obtained naturally from the geometrical structure. The case of Yang–Mills theory was already considered in Loumi and Tahiri (1993). Here we take a step further, and consider a theory with (m, n)-dimensional Lie supergroup G as gauge group and living on the principal superfiber bundle $P_s(M, G_s)$, where the structural supergroup G_s is given by

$$G_s = G \times S^{0,2} \tag{2.1}$$

with the group multiplication

$$(g, \theta, \bar{\theta})(g', \theta', \bar{\theta}') = (gg', \theta + \theta', \bar{\theta} + \bar{\theta}') \tag{2.2}$$

$S^{0,2}$ is the (0, 2)-dimensional vector superspace defined by $S^{0,2} = (B_1)^2$, where B_1 is the odd part of a Grassmann algebra (Rogers, 1981; Bruzzo and Cianci, 1984).

We shall restrict ourselves to a principal superfiber bundle P_s which is globally trivial with respect to $S^{0,2}$. This choice will be related to the fact that the global BRST transformations are the quantum equivalent of the classical gauge invariance.

The Lie superalgebra $\mathcal{G}_s$ of the structural Lie supergroup $G_s = G \times S^{0,2}$ is isomorphic to $\mathcal{G} \oplus s^{0,2}$, where $\mathcal{G}(s^{0,2})$ is the Lie superalgebra of $G(S^{0,2})$. Let I_A ($A = 1, \ldots, m + n$) be the generators of the (m, n)-dimensional Lie supergroup G, and F_α ($\alpha = 1, 2$) the odd generators of $S^{0,2}$. We remark that for Yang–Mills theory $m = d$ and $n = 0$, for gravity $m = 10$ and $n = 0$, and for simple supergravity $m = 10$ and $n = 4$. The generators I_A are even for $A = 1, \ldots, m$ and odd for $A = m + 1, \ldots, m + n$. The Lie superalgebra $\mathcal{G}_s$ is determined by the structure constants

$$[I_A, I_B] = f_{AB}{}^C I_C \tag{2.3a}$$

$$[I_A, F_\alpha] = 0 \tag{2.3b}$$

$$[F_\alpha, F_\beta] = 0 \tag{2.3c}$$

where [,] is the graded Lie bracket.

Let ω be the superconnection in P_s defined as an even $\mathcal{G}_s$-valued 1-superform such that (Rogers, 1981; Bruzzo and Cianci, 1984) (i) $\omega(\tilde{X}) = X$, where $X \in \mathcal{G}_s$ and $\tilde{X}$ is the fundamental vector superfield in P_s associated with X, and (ii) we have

$$R_g^* \omega = \mathrm{ad}(g^{-1})\omega, \qquad g \in G_s \tag{2.4}$$

The supercurvature Ω is an even $\mathcal{G}_s$-valued 2-superform defined from the superconnection in the usual way, satisfying the structure equation

$$\Omega = D\omega = d\omega + 1/2[\omega, \omega] \tag{2.5}$$

where the exterior differential is defined as an even linear map in the space of differential superforms satisfying

$$d(\alpha \wedge \beta) = \alpha \wedge d\beta + (-1)^{\deg \beta}\, d\alpha \wedge \beta \tag{2.6a}$$

$$d^2 = 0 \tag{2.6b}$$

$$df = dZ^a\, \partial_a f \tag{2.6c}$$

where

$$Z = (Z^a)_{a=1,\dots,m+n+6} = (z^M, y^i)_{M=1,\dots,6;i=1,\dots,m+n}$$
$$= (x^\mu, \theta, \bar{\theta}, y^i)_{\mu=1,\dots,4;i=1,\dots,m+n}$$

is a local coordinate system on the total space P_s. We recall that the total space P_s is locally the product of the base space M (the space-time) with the fiber $G_s = G \times S^{0,2}$ (the structure supergroup).

In the local coordinate system $Z = (Z^a)$, we express the superconnection ω in the total space P_s as

$$\omega = dZ^a\omega_a = dz^M\omega_M + dy^i\omega_i = dx^\mu\Phi_\mu + d\theta\phi + d\overline{\theta\phi} + dy^i\omega_i \tag{2.7}$$

By using (2.5) and (2.6), we determine the component superfields of the supercurvature $\Omega = 1/2 dZ^b \wedge dZ^a\Omega_{ab}$:

$$\Omega_{\mu\nu} = \partial_\mu\Phi_\nu - \partial_\nu\Phi_\mu + [\Phi_\mu, \Phi_\nu] \tag{2.8a}$$

$$\Omega_{M\theta} = \partial_M\phi - (-1)^{|M|}\partial_\theta\omega_M + [\omega_M, \phi] \tag{2.8b}$$

$$\Omega_{M\bar{\theta}} = \partial_M\bar{\phi} - (-1)^{|M|}\partial_{\bar{\theta}}\omega_M + [\omega_M, \bar{\phi}] \tag{2.8c}$$

$$\Omega_{Mi} = \partial_M\omega_i - (-1)^{|M|\cdot|i|}\partial_i\omega_M + [\omega_M, \omega_i] \tag{2.8d}$$

$$\Omega_{ij} = \partial_i\omega_j - (-1)^{|i|\cdot|j|}\partial_j\omega_i + [\omega_i, \omega_j] \tag{2.8e}$$

where $|M|$ and $|i|$ represent the Grassmann grades of ω_M and ω_i, respectively.

2.2. BRST and Anti-BRST Transformations

In the superspace formulation of the BRST symmetry in quantized gauge theories, many auxiliary fields appear as components of the superfields, which are eliminated by imposing the vanishing of the supercurvature components along some anticommuting direction.

It is straightforward to obtain these constraints in our approach by the Cartan–Maurer structural theorem, which says

$$i(X)\Omega = 0 \tag{2.9}$$

where i denotes the contraction of vectors with forms and X is a vertical vector superfield in P_s.

To have an adequate expression of (2.9) in coordinates, we write the supercurvature Ω in the $(Z^a) = (z^M, y^i)$ local coordinate system as

$$\Omega = 1/2 dz^N \wedge dz^M \Omega_{MN} + 1/2 dy^i \wedge dz^M \Omega_{Mi} + 1/2 dy^j \wedge dy^i \Omega_{ij} \quad (2.10)$$

and we remark that ∂_θ, $\partial_{\bar{\theta}}$, and ∂_i are vertical vector superfields. Thus

$$\Omega_{M\theta} = \Omega_{M\bar{\theta}} = 0 \quad (2.11a)$$

$$\Omega_{Mi} = \Omega_{ij} = 0 \quad (2.11b)$$

Furthermore, for the BRST and anti-BRST transformations of gauge and ghost fields, we know from the Yang–Mills case that we first consider their variations under the gauge group and then replace the transformation parameters with the corresponding ghosts and a constant anticommuting parameter.

In our case, applying the replacement

$$y^i = \Lambda^i \theta + \overline{\Lambda}^i \bar{\theta} \quad (2.12)$$

we can show that the equations (2.11b), which use the indices of the gauge group coordinates, may be related to the equations (2.11a) as follows: The Grassmann grades of the constant parameters Λ^i and $\overline{\Lambda}^i$ are given by $|\Lambda^i| = |\overline{\Lambda}^i| = |i| + 1 \pmod 2$. The vertical vector fields ∂_i are given by

$$\partial_i = \Lambda_i \partial_\theta + \overline{\Lambda}_i \partial_{\bar{\theta}} \quad (2.13)$$

where $\Lambda^i \Lambda_j + \overline{\Lambda}^i \overline{\Lambda}_j = (-1)^{|j|+1} \delta^i_j$ such that $dy^i(\partial_j) = \delta^i_j$. We then obtain

$$\omega_i = \overline{\Lambda}_i \phi + \Lambda_i \bar{\phi} \quad (2.14)$$

According to (2.8), (2.13), and (2.14), we find

$$\Omega_{Mi} = (-1)^{|M|(|i|+1)} (\Lambda_i \Omega_{M\theta} + \overline{\Lambda}_i \Omega_{M\bar{\theta}}) \quad (2.15a)$$

$$\Omega_{ij} = (-1)^{|j|+1} \{\Lambda_i \Lambda_j \Omega_{\theta\theta} + \overline{\Lambda}_i \overline{\Lambda}_j \Omega_{\bar{\theta}\bar{\theta}} + (\Lambda_i \overline{\Lambda}_j + \overline{\Lambda}_i \Lambda_j) \Omega_{\theta\bar{\theta}}\} \quad (2.15b)$$

Thus, equations (2.11a) and (2.11b) are equivalent under the above replacements. We shall restrict ourselves to the interpretation of (2.11a).

The superconnection and supercurvature components are $\mathcal{G}_s$-valued superfields

$$\omega_M = \omega_M{}^A I_A + \omega_M{}^\alpha F_\alpha \quad (2.16a)$$

$$\Omega_{MN} = \Omega_{MN}{}^A I_A + \Omega_{MN}{}^\alpha F_\alpha \quad (2.16b)$$

In view of (2.3), (2.8), (2.11a), and (2.16), we have

$$\Omega_{\theta\theta}{}^A = 2\partial_\theta \phi - (-1)^{|C|} \phi^C \phi^B f_{BC}{}^A = 0 \quad (2.17a)$$

$$\Omega_{\bar{\theta}\bar{\theta}}{}^A = 2\partial_{\bar{\theta}} \bar{\phi} - (-1)^{|C|} \bar{\phi}^C \bar{\phi}^B f_{BC}{}^A = 0 \quad (2.17b)$$

$$\Omega_{\theta\bar{\theta}}{}^A = \partial_\theta\bar{\phi}^A + \partial_{\bar{\theta}}\phi^A - (-1)^{|C|}\bar{\phi}^C\phi^B f_{BC}{}^A = 0 \tag{2.17c}$$

$$\Omega_{\mu\theta}{}^A = \partial_\mu\phi^A - \partial_\theta\Phi_\mu{}^A + \phi^C\Phi_\mu{}^B f_{BC}{}^A = 0 \tag{2.17d}$$

$$\Omega_{\mu\bar{\theta}}{}^A = \partial_\mu\bar{\phi}^A - \partial_{\bar{\theta}}\Phi_\mu{}^A + \bar{\phi}^C\Phi_\mu{}^B f_{BC}{}^A = 0 \tag{2.17e}$$

$$\Omega_{M\theta}{}^\alpha = \partial_M\phi^\alpha - (-1)^{|M|}\partial_\theta\omega_M{}^\alpha = 0 \tag{2.17f}$$

$$\Omega_{M\bar{\theta}}{}^\alpha = \partial_M\bar{\phi}^\alpha - (-1)^{|M|}\partial_{\bar{\theta}}\omega_M{}^\alpha = 0 \tag{2.17g}$$

where $|C|(|C| + 1)$ is the Grassmann grade of $I_C(\phi^C, \bar{\phi}^C)$ ($|C| = 0$ if $C = 1, \ldots, m$ and $|C| = 1$ if $C = m + 1, \ldots, m + n$).

Two interesting remarks are in order. First, the equations (2.17) with $|C| = 0$ correspond to the constraints of the supercurvature in the context of Yang–Mills theory (Loumi and Tahiri, 1993). Second, the equations (2.17f) and (2.17g) for the potentials ω^α (i.e., ϕ^α, $\bar{\phi}^\alpha$, and $\Phi_\mu{}^\alpha$) associated to F_α correspond to the vanishing of the $S^{0,2}$-supertorsion,

$$\Omega_{MN}{}^\alpha = \partial_M\omega_N{}^\alpha - (-1)^{|M|\cdot|N|}\partial_N\omega_M{}^\alpha \tag{2.18}$$

along some anticommuting direction. We note that this is also the case for Yang–Mills theory (Loumi and Tahiri, 1993). Moreover, in order to remove the $S^{0,2}$-supertorsion dependence of the theory, we supplement the equations (2.17f) and (2.17g) with the constraint

$$\Omega_{\mu\nu}{}^\alpha = \partial_\mu\Phi_\nu{}^\alpha - \partial_\nu\Phi_\mu{}^\alpha = 0 \tag{2.19}$$

Thus

$$\Omega^\alpha = 0 \tag{2.20}$$

The potentials ω^α being a pure gauge because of (2.20), we will concentrate on the transformation laws for ω^A (i.e., ϕ^A, $\bar{\phi}^A$, and $\Phi_\mu{}^A$).

We expand $\omega^B(x^\mu, \theta, \bar{\theta})$ in power series of θ and $\bar{\theta}$:

$$\Phi_\mu{}^B(x, \theta, \bar{\theta}) = A_\mu{}^B(x) + \theta R_\mu{}^B(x) + \bar{\theta}\bar{R}_\mu{}^B(x) + \theta\bar{\theta}S_\mu{}^B(x) \tag{2.21a}$$

$$\phi^B(x, \theta, \bar{\theta}) = c^B(x) + \theta r^B(x) + \bar{\theta}\bar{r}^B(x) + \theta\bar{\theta}s^B(x) \tag{2.21b}$$

$$\bar{\phi}^B(x, \theta, \bar{\theta}) = \bar{c}^B(x) + \theta\bar{t}^B(x) + \bar{\theta}t^B(x) + \theta\bar{\theta}u^B(x) \tag{2.21c}$$

Equations (2.17a)–(2.17e) are expressed in terms of the field components of the superconnection by

$$R_\mu{}^B = \partial_\mu c^B + c^D A_\mu{}^E f_{ED}{}^B \tag{2.22a}$$

$$\bar{R}_\mu{}^B = \partial_\mu\bar{c}^B + \bar{c}^D A_\mu{}^E f_{ED}{}^B \tag{2.22b}$$

$$r^B = 1/2(-1)^{|D|}c^D c^E f_{ED}{}^B \tag{2.22c}$$

$$t^B = 1/2(-1)^{|D|}\bar{c}^D\bar{c}^E f_{ED}{}^B \tag{2.22d}$$

$$\bar{t}^B + \bar{r}^B - (-1)^{|D|}\bar{c}^D c^E f_{ED}{}^B = 0 \tag{2.22e}$$

$$S_\mu{}^B = \partial_\mu \bar{r}^B + \bar{r}^D A_\mu{}^E f_{EB}{}^B - (-1)^{|D|}c^D \bar{R}_\mu{}^E f_{ED}{}^B \tag{2.22f}$$

$$s^B = (-1)^{|D|}\bar{r}^D c^E f_{ED}{}^B \tag{2.22g}$$

$$u^B = -(-1)^{|D|}\bar{t}^D\bar{c}^E f_{ED}{}^B \tag{2.22h}$$

Let Q and $\bar{Q}$ be the differential operators representing the $S^{0,2}$-generators F_α ($\alpha = 1, 2$). The $S^{0,2}$-motion in the total space P_s

$$R(\theta, \bar{\theta}): \quad (x^\mu, \xi, \bar{\xi}) \rightarrow (x^\mu, \xi + \theta, \bar{\xi} + \bar{\theta}) \tag{2.23}$$

may be generated by $\theta Q + \overline{\theta Q}$. Thus, the operational representation is given by

$$r(\theta, \bar{\theta}) = \exp(\theta Q) + \overline{\theta Q}) \tag{2.24}$$

For infinitesimal group action and by using Hausdorff's formula (all commutators vanish because $[Q, Q] = [\bar{Q}, \bar{Q}] = [Q, \bar{Q}] = 0$), we have

$$r(\theta, \bar{\theta}) = (1 + \theta Q)(1 + \bar{\theta}\bar{Q}) \tag{2.25}$$

According to (2.4), (2.23), and (2.25), we find

$$\begin{aligned}
&R^*(\theta, \bar{\theta})\omega_M{}^A(x^\mu, \xi, \bar{\xi}) \\
&\quad = \omega_M{}^A(x^\mu, \xi + \theta, \bar{\xi} + \bar{\theta}) \\
&\quad = r(\theta, \bar{\theta})\omega_M{}^A(x^\mu, \xi, \bar{\xi})r^{-1}(\theta, \bar{\theta}) \\
&\quad = \omega_M{}^A(x^\mu, \xi, \bar{\xi}) + \theta[Q, \omega_M{}^A(x^\mu, \xi, \bar{\xi})] \\
&\qquad + \bar{\theta}[\bar{Q}, \omega_M{}^A(x^\mu, \xi, \bar{\xi})] \\
&\qquad + \theta\bar{\theta}[\bar{Q}, [Q, \omega_M{}^A(x^\mu, \xi, \bar{\xi})]]
\end{aligned} \tag{2.26}$$

where we have used

$$[Q, [\bar{Q}, \omega_M{}^A(x^\mu, \xi, \bar{\xi})]] = -[\bar{Q}, [Q, \omega_M{}^A(x^\mu, \xi, \bar{\xi})]]$$

Evaluating (2.26) at $\xi = \bar{\xi} = 0$ gives

$$\Phi_\mu{}^B = A_\mu{}^B + \theta[Q, A_\mu{}^B] + \bar{\theta}[\bar{Q}, A_\mu{}^B] + \theta\bar{\theta}[\bar{Q}, [Q, A_\mu{}^B]] \tag{2.27a}$$

$$\phi^B = c^B + \theta[Q, c^B] + \bar{\theta}[\bar{Q}, c^B] + \theta\bar{\theta}[\bar{Q}, [Q, c^B]] \tag{2.27b}$$

$$\bar{\phi}^B = \bar{c}^B + \theta[Q, \bar{c}^B] + \bar{\theta}[\bar{Q}, \bar{c}^B] + \theta\bar{\theta}[\bar{Q}, [Q, \bar{c}^B]] \tag{2.27c}$$

From (2.22), we get the minimum number of independent component fields: $A_\mu{}^D$, c^D, $\bar{c}^D$, $\bar{t}^D \equiv B^D$.

Using (2.21), (2.22), and (2.27), we obtain

$$[Q, A_\mu{}^D] = \partial_\mu c^D + c^F A_\mu{}^E f_{EF}{}^D \tag{2.28a}$$

$$[Q, c^D] = 1/2(-1)^{|F|} c^F c^E f_{EF}{}^D \tag{2.28b}$$

$$[Q, \bar{c}^D] = B^D \tag{2.28c}$$

$$[Q, B^D] = 0 \tag{2.28d}$$

and

$$[\bar{Q}, A_\mu{}^D] = \partial_\mu \bar{c}^D + \bar{c}^F A_\mu{}^E f_{EF}{}^D \tag{2.29a}$$

$$[\bar{Q}, \bar{c}^D] = 1/2(-1)^{|F|} \bar{c}^F \bar{c}^E f_{EF}{}^D \tag{2.29b}$$

$$[\bar{Q}, c^D] = B'^D \tag{2.29c}$$

$$[\bar{Q}, B'^D] = 0 \tag{2.29d}$$

where

$$\bar{r}^D \equiv B'^D = -B^D + (-1)^{|F|} \bar{c}^F c^E f_{EF}{}^D \tag{2.30}$$

In the case of Yang–Mills theory, the relations (2.28)–(2.30) reproduce the BRST and anti-BRST transformations, where A_μ, c, $\bar{c}$, B, Q, and $\bar{Q}$ represent the potential, ghost, antighost, auxiliary field, and charges of the BRST and anti-BRST transformations, respectively (Becchi *et al.*, 1975; Tyutin, 1975; Curci and Ferrari, 1976; Ojima, 1980; Bonora and Tonin, 1981; Bonora *et al.*, 1982; Hoyos *et al.*, 1982, 1983; Loumi and Tahiri, 1993).

3. THE CASE OF GRAVITY IN THE VIERBEIN FORMALISM

In this section we apply the general formalism of the previous subsection to gravity in the vierbein formalism. The gauge group G is the Poincaré group P generated by M_{ab} and P_a satisfying the following commutators:

$$[M_{ab}, M_{cd}]_- = f_{ab,cd}{}^{gh} M_{gh} \tag{3.1a}$$

$$[M_{ab}, P_c]_- = f_{ab,c}{}^d P_d \tag{3.1b}$$

$$[P_a, P_b]_- = 0 \tag{3.1c}$$

where

$$f_{ab,cd}{}^{gh} = 1/2(\eta_{ad}\delta_b{}^{[g}\delta_c{}^{h]} + \eta_{bc}\delta_a{}^{[g}\delta_d{}^{h]} - \eta_{ac}\delta_b{}^{[g}\delta_d{}^{h]} - \eta_{bd}\delta_a{}^{[g}\delta_c{}^{h]}) \tag{3.1d}$$

$$f_{ab,c}{}^d = 1/2(\eta_{bc}\delta_a{}^d - \eta_{ac}\delta_b{}^d) \tag{3.1e}$$

The gauge field $\omega_\mu{}^{ab}$ (spin-affine connection) is determined by the vierbein components $e_\mu{}^a$,

$$\omega_\mu{}^{ab} = -e^{\nu b}(\partial_\mu e_\nu{}^a - \Gamma_{\mu\nu}{}^\lambda e_\lambda{}^a) \tag{3.2}$$

According to (2.28), (2.29), and (3.1), we find

$$[Q, \omega_\mu{}^{ab}]_- = \partial_\mu c^{ab} + c_f{}^b \omega_\mu{}^{af} - c^a{}_f \omega_\mu{}^{fb} \tag{3.3a}$$

$$[Q, e_\mu{}^a]_- = -c^a{}_f e_\mu{}^f \tag{3.3b}$$

$$[Q, c^{ab}]_+ = -c^b{}_f c^{fa} \tag{3.3c}$$

$$[Q, \bar{c}^{ab}]_+ = B^{ab} \tag{3.3d}$$

$$[Q, B^{ab}]_- = 0 \tag{3.3e}$$

and

$$[\bar{Q}, \omega_\mu{}^{ab}]_- = \partial_\mu \bar{c}^{ab} + \bar{c}_f{}^b \omega_\mu{}^{af} - \bar{c}^a{}_f \omega_\mu{}^{fb} \tag{3.4a}$$

$$[\bar{Q}, e_\mu{}^a]_- = -\bar{c}^a{}_f e_\mu{}^f \tag{3.4b}$$

$$[\bar{Q}, \bar{c}^{ab}]_+ = -\bar{c}^b{}_f \bar{c}^{fa} \tag{3.4c}$$

$$[\bar{Q}, c^{ab}]_+ = B'^{ab} \tag{3.4d}$$

$$[\bar{Q}, B'^{ab}]_- = 0 \tag{3.4e}$$

where

$$B'^{ab} = -B^{ab} - \bar{c}^a{}_d c^{db} + \bar{c}^b{}_d c^{da} \tag{3.5}$$

In (3.3) and (3.4), we have used the antisymmetry of Latin indices, for instance,

$$c^{ab} = -c^{ba}, \qquad c^a{}_b = \eta_{bd} c^{ad} = -\eta_{bd} c^{da} = -c_b{}^a$$

The relations (3.3) and (3.4) give the BRST and anti-BRST transformations of the spin-affine connection $\omega_\mu{}^{ab}$, the vierbein $e_\mu{}^a$, the auxiliary field B^{ab}, and the local Lorentz ghosts and antighosts c^{ab} and $\bar{c}^{ab}$. They represent the symmetry of the local Lorentz part of quantum gravity in the vierbein formalism (Nakanishi, 1979; Ojima, 1980).

4. THE CASE OF SIMPLE SUPERGRAVITY

Simple supergravity without auxiliary fields is described in the fiber bundle formalism when one uses a principal superfiber bundle whose structural group is the super-Poincaré group *SP* (Yates, 1980). By denoting the generators M_{ab}, P_a, and Q_α, we have that the Lie superalgebra of *SP* is given by (3.1) supplemented with

$$[M_{ab}, Q_\alpha]_- = f_{ab,\alpha}{}^\beta Q_\beta \tag{4.1a}$$

$$[Q_\alpha, Q_\beta]_+ = f_{\alpha,\beta}{}^a P_a \tag{4.1b}$$

$$[Q_\alpha, P_a]_- = 0 \tag{4.1c}$$

where

$$f_{ab,\alpha}{}^\beta = 1/2(\sigma_{ab})_\alpha{}^\beta \tag{4.1d}$$

$$f_{\alpha,\beta}{}^a = (C\gamma^a)_{\alpha\beta} \tag{4.1e}$$

C is the charge conjugation matrix, γ the Dirac matrices, and $\sigma_{ab} = 1/4[\gamma_a, \gamma_b]_-$.

The only fields arising in the superfiber bundle approach to simple supergravity are the vierbein $e_\mu{}^a$, the spin-3/2 field $\psi_\mu{}^\alpha$, and the connection $\omega_\mu{}^{ab}$, which is given by (Yates, 1980)

$$\begin{aligned}\omega_\mu{}^{ab} = 1/2\{& e^{\nu b}(\partial_\nu e_\mu{}^a - \partial_\mu e_\nu{}^a) - e^{\nu a}(\partial_\nu e_\mu{}^b - \partial_\mu e_\nu{}^b) \\ &+ e_\mu{}^c e^{\lambda a} e^{\nu b}(\partial_\nu e_{c\lambda} - \partial_\lambda e_{c\nu}) + e^{\nu b}\psi_\nu C\gamma^a\psi_\mu - e^{\nu a}\psi_\nu C\gamma^b\psi_\mu \\ &+ e_{c\mu}e^{\lambda a}e^{\nu b}\psi_\nu C\gamma^c\psi_\lambda - e^{\nu b}T_{\nu\mu}{}^a + e^{\nu a}T_{\nu\mu}{}^b - e_{c\mu}e^{\lambda a}e^{\nu b}T_{\nu\lambda}{}^c\}\end{aligned} \tag{4.2}$$

where

$$T_{\mu\nu}{}^a = \partial_\mu e_\nu{}^a - \partial_\nu e_\mu{}^a + \omega_\mu{}^{ab}e_{b\nu} - \omega_\nu{}^{ab}e_{b\mu} + \psi_\mu C\gamma^a\psi_\nu \tag{4.3}$$

We remark that $\omega_\mu{}^{ab}$ is an obvious generalization of the spin-affine connection. Let c^{ab}, $\bar{c}^{ab}$ and $\mathbf{c}^\alpha$, $\bar{c}^\alpha$ be the local Lorentz ghosts and the local supersymmetry ghosts associated with the Lorentz generators M_{ab} and the supersymmetry generators Q_α, respectively. The complex tensor c^{ab} is anticommuting with itself, but the complex four-component spin-1/2 ghost c^α is commuting with itself. Using (2.28), (2.29), (3.1), and (4.1), we find that the BRST and anti-BRST transformations for simple supergravity (Van Nieuwenhuizen (1981)) of the connection $\omega_\mu{}^{ab}$, the vierbein $e_\mu{}^a$, the gravitino $\psi_\mu{}^\alpha$, the local Lorentz ghosts c^{ab}, $\bar{c}^{ab}$, and the local supersymmetry ghosts c^α, $\bar{c}^\alpha$ are given by (3.3) and (3.4) supplemented with

$$[Q, \psi_\mu{}^\alpha]_+ = \partial_\mu c^\alpha - 1/2c^{ab}\psi_\mu{}^\beta(\sigma_{ab})_\beta{}^\alpha \tag{4.4a}$$

$$[Q, c^\alpha]_- = -1/2c^\beta c^{ab}(\sigma_{ab})_\beta{}^\alpha \tag{4.4b}$$

$$[Q, \bar{c}^\alpha]_- = B^\alpha \tag{4.4c}$$

$$[Q, B^\alpha]_+ = 0 \tag{4.4d}$$

and

$$[\bar{Q}, \psi_\mu{}^\alpha]_+ = \partial_\mu\bar{c}^\alpha - 1/2\bar{c}^{ab}\psi_\mu{}^\beta(\sigma_{ab})_\beta{}^\alpha \tag{4.5a}$$

$$[\overline{Q}, \bar{c}^{\alpha}]_- = -1/2\bar{c}^{\beta}\bar{c}^{ab}(\sigma_{ab})_{\beta}{}^{\alpha} \tag{4.5b}$$

$$[\overline{Q}, c^{\alpha}]_- = B'^{\alpha} \tag{4.5c}$$

$$[\overline{Q}, B'^{\alpha}]_+ = 0 \tag{4.5d}$$

where

$$B'^{\alpha} = -B^{\alpha} - 1/2(\bar{c}^{\beta}\bar{c}^{ab} + \bar{c}^{ab}c^{\beta})(\sigma_{ab})_{\beta}{}^{\alpha} \tag{4.6}$$

5. DISCUSSION

We have seen that it is possible to construct a geometrical model for quantized gauge theories, based on a Lie supergroup as gauge group, by using a superfiber bundle with base simply space-time and structure group the product of the gauge group with the two-dimensional odd translation group. In this geometrical framework, the soul-flatness condition is naturally obtained by the fact that the supercurvature is a tensorial superform. Then the BRST and anti-BRST transformations are determined by using the pseudo-tensoriality of the superconnection in the adjoint operational representation. So we have a geometrical formulation for gauge theories with Faddeev–Popov fields which is directly related to the superfiber bundle formalism without imposing constraints as in the superspace formulation.

We have also seen that the BRST and anti-BRST transformations for gravity and simple supergravity can be cast into the same form by choosing the gauge group as the Poincaré or super-Poincaré group, respectively. Application to simple supergravity leads only to local Lorentz ghosts and local supersymmetry ghosts. It would be interesting to describe in the same way the general coordinate ghosts. This can be performed via local $Osp(4|2)$ supersymmetry as in the case of gravity (Delbourgo *et al.*, 1982).

Moreover, we should remark that the gauge-fixing plus ghost Lagrangian L^{gf} of the spin-affine connection part of quantum gravity in the vierbein formalism (Nakanishi, 1979; Ojima, 1980) can be represented as a super-Lagrangian L_s^{gf} as follows: Loumi and Tahiri (1993) established that the gauge-fixing plus ghost super-Lagrangian of Yang–Mills theory is given by

$$L_s^{gf} = (\partial^{\mu}\Phi_{\mu})(\partial_{\theta}\bar{\phi}) + (\partial^{\mu}\bar{\phi})(\partial_{\theta}\Phi_{\mu}) + 1/2\alpha\{(\partial_{\theta}\bar{\phi})^2 + (\partial_{\bar{\theta}}\phi)^2\} \tag{5.1}$$

We can here easily show that this super-Lagrangian represents also that of the local Lorentz part of quantum gravity. Indeed, since the gauge-fixing plus ghost Lagrangian L^{gf} is given by the lowest component of L_s^{gf}, and $\omega_{\mu}{}^{ab}$, c^{ab}, $\bar{c}^{ab}$, B^{ab}, and B'^{ab} are, respectively, the (ab)-components of the lowest components of Φ_{μ}, ϕ, $\bar{\phi}$, $\partial_{\theta}\bar{\phi}$, and $\partial_{\bar{\theta}}\phi$, we get

$$L^{gf} = B_{ab}\partial^{\mu}\omega_{\mu}{}^{ab} + \partial^{\mu}\bar{c}_{ab}(\partial_{\mu}c^{ab} + \eta_{cd}\omega_{\mu}{}^{ca}c^{bd} - \eta_{cd}\omega_{\mu}{}^{cb}c^{ad})$$
$$+ 1/2\alpha(B_{ab}B^{ab} + B'_{ab}B'^{ab}) \qquad (5.2)$$

This represents precisely the gauge-fixing plus ghost Lagrangian of the spin-affine connection part of quantum gravity in the vierbein formalism (Nakanishi, 1979; Ojima, 1980).

Finally, we should also remark that simple supergravity without auxiliary fields is a theory with an open algebra. We recall that the only fields arising in the superfiber bundle approach to simple supergravity are the vierbein $e_{\mu}{}^{a}$, the gravitino $\psi_{\mu}{}^{\alpha}$, and the connection $\omega_{\mu}{}^{ab}$. On the other hand, the gauge theories whose gauge algebra is open (or/and which are reducible) can be quantized by the Batalin–Vilkovisky formalism, which provides gauge-fixed actions with on-shell nilpotent BRST transformations (Batalin and Vilkovisky, 1981, 1983). In this framework, the quantization of supergravity has been realized by Baulieu *et al.* (1990), while Antunović *et al.* (1993) proposed a generalization of the Batalin–Vilkovisky formalism so that the BRST structure of simple supergravity becomes nilpotent off-shell via the introduction of auxiliary fields. It would be interesting to find out how to make the BRST and anti-BRST algebra of simple supergravity obtained in the present work nilpotent off-shell. This may be realized by enlarging the space of fields in the theory by auxiliary fields, which can be naturally introduced through the supercurvature. We have used similar arguments for topological antisymmetric tensor gauge theories, so-called BF theories (Blau and Thompson, 1991), in four dimensions (Tahiri, 1994). Here, the supercurvature associated to the superconnection representing the antisymmetric tensor gauge field and its hierarchy of ghosts permits the natural introduction of auxiliary fields, which lead to the construction of an off-shell nilpotent BRST and anti-BRST algebra.

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